CBCS Scheme

USN | | | | | | | | | | 15CS36

Third Semester B.E. Degree Examination, Dec.2017/Jan.2018

Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Prove that for any three propositions p, q, r $[P \to (q \wedge r)] \Leftrightarrow [(p \to q) \wedge (p \to r)]$. Using truth table. **(05 Marks)**

b. Establish the validity of the argument :

$p \to q$
$q \to (r \wedge s)$
$\neg r \vee (\neg t \vee u)$
$p \wedge t$
$\therefore u$ **(06 Marks)**

c. Prove that for all integers 'k' and 'ℓ', if 'k' and 'ℓ' are both odd, then $k + \ell$ is even and $k\ell$ is odd by direct proof. **(05 Marks)**

OR

2 a. Determine the truth value of each of the following quantified statements; the universe being the set of all non - zero integers. **(05 Marks)**

i) $\exists x, \exists y\ [xy = 1]$
ii) $\exists x, \forall y\ [xy = 1]$
iii) $\forall x, \exists y,\ [xy = 1]$
iv) $\exists x, \exists y\ [(2x + y = 5) \wedge (x - 3y = -8)]$.
v) $\exists x, \exists y\ [(3x - y = 17) \wedge (2x + 4y = 3)]$. **(06 Marks)**

b. Find whether the following arguments are valid or not for which the universe is set of all triangles. In triangle XYZ, there is no pair of angles of equal measure. If the triangle has two sides of equal length, then it is isosceles. If the triangle is isosceles, then it has two angles of equal measure. Therefore triangle XYZ has no two sides of equal length. **(05 Marks)**

c. If a proposition has truth value 1, determine all truth value assignments for the primitive propositions p, r, s for which the truth value of following compound proposition is 1.
$[q \to \{(\neg p \vee r) \wedge \neg s\}] \wedge \{\neg s \to (\neg r \wedge q)\}$. **(05 Marks)**

Module-2

3 a. Prove by mathematical induction that, for every positive integer n, 5 divides $n^5 - n$. **(05 Marks)**

b. For the Fibonacci sequence F_0, F_1, F_2 - - - - prove that $F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$. **(06 Marks)**

c. Find the coefficient of :
i) x^9y^3 in the expansion $(2x - 3y)^{12}$
ii) x^{12} in the expansion $x^3(1 - 2x)^{10}$. **(05 Marks)**

OR

4 a. By mathematical induction. Prove that, for every positive integer n, the number $A_n = 5^n + 2.3^{n-1} + 1$ is a multiple of 8. **(05 Marks)**

b. How many positive integers 'n' can we form using the digits 3, 4, 4, 5, 5, 6, 7 if we want 'n' to exceed 5,000,000. **(06 Marks)**

c. A certain question paper contains three parts A, B, C with four questions in part A, five questions in part B and six questions in part C. It is required to answer seven questions selecting atleast two questions from each part. In how many ways can a student select his seven questions for answering? **(05 Marks)**

Module-3

5 a. Let $f : R \to R$ be defined by $f(x) = \begin{cases} 3x-5, & \text{for } x > 0 \\ -3x+1, & \text{for } x \le 0 \end{cases}$

i) Determine $f\left(\frac{5}{3}\right)$, $f^{-1}(3)$, $f^{-1}([-5, 5])$.

ii) Also prove that if 30 dictionaries contain a total of 61, 327 pages, then atleast one of the dictionary must have atleast 2045 pages. **(05 Marks)**

b. Prove that if $f : A \to B$ and $g : B \to C$ are invertible function then $g \circ f : A \to C$ is an invertible function and $(gof)^{-1} = f^{-1}og^{-1}$. **(06 Marks)**

c. Let A = {1, 2, 3, 4, 5}. Define a relation R on A × A by $(x_1, y_1)\ R(x_2, y_2)$ if and only if $x_1 + y_1 = x_2 + y_2$.

i) Determine whether R is an equivalence relation on A × A

ii) Determine equivalence class [(1, 2)], [(2, 5)]. **(05 Marks)**

OR

6 a. Let f and g be functions from R to R defined by f(x) = ax + b and $g(x) = 1 - x + x^2$. If $(g \circ f)(x) = 9x^2 - 9x + 3$. Determined a, b. **(05 Marks)**

b. Let A = {1, 2, 3, 4, 6, 12}. On A define the relation R by aRb if and only if 'a' divides 'b' i) prove that R is a partial order on A ii) draw the Hasse diagram iii) write down the matrix of relation. **(06 Marks)**

c. Consider the Poset whose Hasse diagram is given below. Consider B = {3, 4, 5}. Refer Fig.Q6(c). Find :

i) All upper bounds of B

ii) All lower bounds of B

iii) The least upper bound of B

iv) The greatest lower bound of B

v) Is this a Lattice? **(05 Marks)**

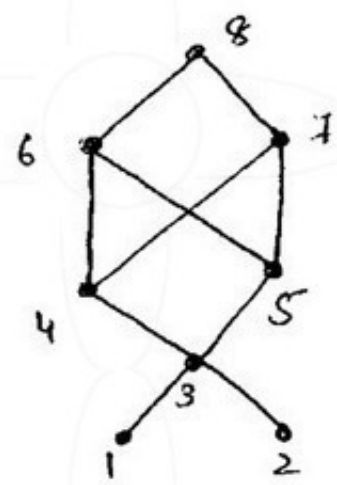

Fig.Q6(c)

Module-4

7 a. Out of 30 students in a hostel; 15 study history 8 study economics and 6 study geography. It is known that 3 students study all these subjects. Show that 7 or more students study none of these subjects. **(05 Marks)**

b. Five teachers T_1, T_2, T_3, T_4, T_5 are to be made class teachers for five classes C_1, C_2, C_3, C_4, C_5, one teacher for each class. T_1 and T_2 do not wish to become the class teachers for C_1 or C_2, T_3 and T_4 for C_4 or C_5 and T_5 for C_3 or C_4 or C_5. In how many ways can the teachers be assigned work without displeasing any teacher. **(06 Marks)**

c. Solve the recurrence relation $a_n - 6a_{n-1} + 9a_{n-2} = 0$ form $n \geq 2$. **(05 Marks)**

OR

8 a. Solve the recurrence relation $a_n - 3a_{n-1} = 5 \times 3^n$ for $n \geq 1$ given that $a_0 = 2$. **(05 Marks)**

b. Let a_n denote the number of n-letter sequences that can be formed using the letters A, B and C such that non terminal A has to be immediately followed by a B. Find the recurrence relation for a_n and solve it. **(06 Marks)**

c. Find the number of permutations of English letters which contain exactly two of the pattern car, dog, pun, byte. **(05 Marks)**

Module-5

9 a. Discuss Konigsberg bridge problem. **(05 Marks)**

b. Let G = G(V, E) be a simple graph with m edges and 'n' vertices. Then prove that :

i) $m \leq \frac{1}{2}n(n-1)$

ii) For a complete graph k_n, $m = \frac{1}{2}n(n-1)$ edges

iii) How many vertices and edges are there for $K_{4,7}$ and $K_{7,11}$. **(06 Marks)**

c. Merge sort the list –1, 7, 4. 11, 5. –8, 15, –3, –2, 6. 10. 3. **(05 Marks)**

OR

10 a. Prove that a tree with 'n' vertices has n – 1 edges. **(05 Marks)**

b. Obtain an optimal prefix code for the message LETTER RECEIVED indicate the code and weight. **(06 Marks)**

c. Determine whether the following graphs are isomorphic or not. **(05 Marks)**

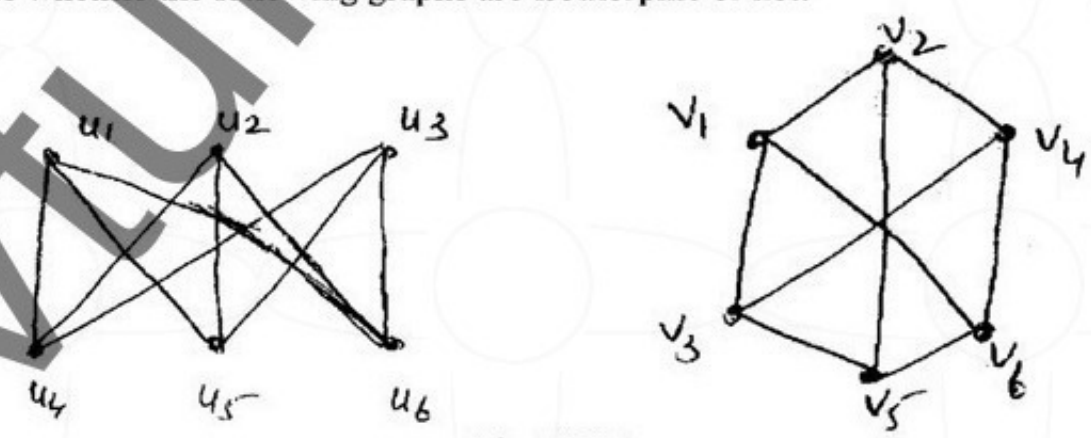

Fig.Q10(c)

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